Computing Derivatives

Math 130 - Essentials of Calculus

7 October 2019

Derivatives and the Information They Provide

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5. $f(x) = \sqrt[5]{x}$
6. $w(z) = \dfrac{1}{\sqrt{z}}$
7. $g(t) = \dfrac{1}{\sqrt[7]{x^{12}}}$

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Derivatives of Sums and Differences

Theorem

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

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e is the number such that

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